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Fuzzy 5-spheres and pp-wave Matrix actions

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Abstract: Using the action describing N coincident gravitational waves in M-theory we construct a pp-wave Matrix model containing a fuzzy 5-sphere giant graviton solution. This fuzzy 5-sphere is constructed as a U(1) fibration over a fuzzy CP^2, and has the correct dependence of the radius with the light-cone momentum, $r^4 \sim p^+$, to approach the 5-sphere giant graviton solution of Mc.Greevy et al in the large N limit.

Keywords: Penrose limit and pp-wave background, M(atrix) Theories, M-Theory.

Contents

1. Introduction

The BMN Matrix model [1] is an example of Matrix theory in a non-trivial background, the maximally supersymmetric pp-wave background of M-theory [2]. Compared to Matrix theory in a flat background it contains additional mass and dielectric terms that remove the flat directions [1, 3], leaving isolated vacua in the form of giant gravitons [4]. String theory in this background is solvable [5], and through the AdS/CFT correspondence, type-IIB string theory in the pp-wave background has been shown to be dual to the, so-called, BMN sector of four dimensional $\mathcal{N}=4$ SYM [1].

The derivation of the BMN Matrix model was based on the generalization to arbitrary number of particles of the action for a superparticle in the pp-wave background, under the requirement of consistency with supersymmetry [1]. The same Matrix model was obtained in [3] by regularizing the light-cone supermembrane action in the pp-wave background, in the same way Matrix theory in a flat background arises from the flat supermembrane of [6] (see also [7]). Moreover, the model was shown to be related to the action for type-IIA D0-branes upon dimensional reduction plus Sen-Seiberg [8, 9] limit.

From the last point of view one could say that the basic degrees of freedom of the BMN Matrix model are gravitational waves propagating in the light-cone direction. An action describing coincident gravitational waves in arbitrary M-theory backgrounds has been constructed in [10]. This action goes beyond the linear order approximation of Matrix theory in a weakly curved background [11] precisely by imposing consistency with the action for coincident type IIA D0-branes upon reduction along the direction of propagation. One non-trivial check of the validity of this action is that it has been successfully used in the

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study of gravitational waves in $AdS_m \times S^n$ backgrounds, which are not linear perturbations to Minkowski, and in particular in the microscopical description of giant gravitons in terms of dielectric gravitational waves[1] [10, 12 − 14].

For consistency the BMN Matrix model should come up as the action of [10] in the maximally supersymmetric pp-wave background of M-theory. Indeed, one can check that when the waves propagate in this background with a non-vanishing light-cone momentum the action of [10] reduces to the BMN Matrix model. In this derivation the dielectric coupling present in the BMN Matrix model simply arises as the non-Abelian Myers coupling [15] to the 3-form potential in the action for the waves. As shown in [1] this coupling supports a zero-energy solution consisting on the expansion of the waves by dielectric effect into a fuzzy 2-sphere with light-cone momentum, which constitutes the microscopical version of the 2-sphere giant graviton solution of [4] in this background.

A 5-sphere giant graviton solution is also known in the M-theory pp-wave background [4], a microscopical description to which has not successfully been given so far. One would expect that microscopically the waves would expand into a fuzzy 5-sphere through their (quadrupolar) coupling to the 6-form potential of the background.[2] However, such a coupling does not exist in the BMN Matrix model, this being the reason why a fuzzy 5-sphere giant graviton solution cannot easily be constructed. This is of course related to the difficulty in seeing the M5-brane in the Matrix model [16].

As one would expect, this problem is already encountered in the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ backgrounds from which the M-theory pp-wave emerges after Penrose limit. In these backgrounds there is a 5-sphere giant graviton solution which expands either into the S^7 or the AdS_7 parts of the geometry[3] [4, 17]. The microscopical description of these giant gravitons in terms of dielectric gravitational waves has been given recently in [14]. In this reference it is shown that in order to find the quadrupolar couplings to the 6-form potential responsible for the expansion of the waves it is necessary to go to a more non-perturbative regime of M-theory. This regime is reached by interchanging the direction of propagation of the waves with another compact direction of the background with a geometrical meaning, that we will clarify below. Reduction along the direction of propagation gives rise to type-IIA D0-branes, which upon T-duality along the geometrical compact direction that they inherit from M-theory give rise to type-IIB D-strings. On the other hand, reduction of the waves along the geometrical compact direction gives rise to type-IIA waves, which upon T-duality along their direction of propagation give rise to type-IIB F-strings. The S-duality between the F-strings and the D-strings is therefore generated in M-theory through the interchange of the direction of propagation and the geometrical compact direction.

[1] Of course, to describe these giant gravitons in non M-theory AdS type backgrounds the analogous action for coincident waves has to be constructed in type II, but this can easily be done using dualities from the M-theory action.

[2] Since a 5-sphere has 5 relative dimensions with respect to a point-like object, the 6-form potential has to be contracted as well with some other (Abelian) direction of the background in order to be able to couple to a one dimensional worldvolume, as we will see.

[3] The giant graviton expanding into the anti-deSitter part of the geometry is referred in the literature as dual giant graviton [17, 18].

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From this point of view the action of [10] describes *perturbative* gravitational waves. The details of this action (see [10] and [19]) show that the direction of propagation of the waves is isometric, being in this sense special. The discussion in the previous paragraph suggests that we can use the same action to describe *non-perturbative* gravitational waves if the isometric direction is instead identified with some compact geometrical direction of the background. Indeed, one can see that doing this in the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ backgrounds one finds non-vanishing quadrupolar couplings to the 6-form potential that can cause the expansion of the waves into fuzzy 5-spheres [14]. These fuzzy 5-spheres are defined as S^1 bundles over fuzzy CP^2, and the direction along the S^1 is the one playing the role of compact geometrical direction. The strongest check of the validity of this action is that the corresponding microscopical giant and dual giant graviton solutions have the same radii and satisfy the same bound (for the giant graviton), for large number of gravitons, than the classical 5-sphere solutions of [4] and [17].

The expansion of the waves into fuzzy 5-spheres is from the previous point of view more non-perturbative than the corresponding expansion into fuzzy 2-spheres. This is in agreement with previous observations along these lines [1, 20]. In these references it is argued that the 5-brane cannot appear as a classical solution to the BMN Matrix model because the scaling of its radius with the coupling constant is more non-perturbative than the one corresponding to a classical solution, and it is compared to the difficulty in realizing classically the process in which N D3-branes blow up into an NS5-brane found in [21] in mass deformed $\mathcal{N} = 4$ SYM. In this case a non-perturbative action for coincident D3-branes would show a dielectric coupling to the NS-NS 6-form potential that would cause the expansion of the D3-branes into an NS5-brane. Our discussion above shows that this difficulty is also present in M-theory, and moreover, that it is not peculiar to the pp-wave background.

In this paper we will use the action of [10] to construct a *non-perturbative* Matrix model which is solved by a non-commutative 5-sphere. We will see that this 5-sphere has the same radius and energy than the classical 5-sphere giant graviton solution of [4] in this background. We will also comment in the conclusions on the relation between our solution and previous attempts in the literature to obtain the fuzzy 5-sphere giant graviton as a supersymmetry preserving solution [22–24].

2. The action for M-theory gravitational waves

The worldvolume theory associated to N coincident gravitational waves in M-theory is a U(N) gauge theory, in which the vector field is associated to M2-branes (wrapped on the direction of propagation of the waves) ending on them [10]. This vector field gives the BI field living in a set of coincident D0-branes upon reduction along the direction of propagation of the waves.

In this paper we will use a truncated version of the action in [10] in which the vector field is set to zero, given that it will not play any role in the backgrounds that we will be discussing. This action is given by $S = S^{\rm BI} + S^{\rm CS}$, with

$$S^{\rm BI} = -\int dt\ {\rm STr}\left\{k^{-1}\sqrt{-P\left[E_{00} + E_{0i}(Q^{-1}-\delta)^i_k E^{kj} E_{j0}\right]\det Q}\right\}, \tag{2.1}$$

where

$$
\begin{aligned}
E_{\mu\nu} &= \mathcal{G}_{\mu\nu} + k^{-1}(\mathrm{i}_k C^{(3)})_{\mu\nu}\,, \qquad \mathcal{G}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \\
Q^i_j &= \delta^i_j + ik[X^i, X^k]E_{kj}\,,
\end{aligned}
\tag{2.2}
$$

and

$$
S^{\mathrm{CS}} = \int dt\ \mathrm{STr}\left\{ -P[k^{-2}k^{(1)}] + iP\left[(\mathrm{i}_X\mathrm{i}_X)C^{(3)}\right] + \frac{1}{2}P\left[(\mathrm{i}_X\mathrm{i}_X)^2\mathrm{i}_k C^{(6)}\right] + \cdots \right\}, \tag{2.3}
$$

where the dots include couplings to higher order background potentials and products of different background fields contracted with the non-abelian scalars[4]. We have also taken T_0, the tension of a single gravitational wave, equal to one. In this action k^μ is an abelian Killing vector which, by construction, points on the direction of propagation of the waves. This direction is isometric, because the background fields are either contracted with the Killing vector, so that any component along the isometric direction of the contracted field vanishes, or pulled back in the worldvolume with covariant derivatives relative to the isometry (see [10] for their explicit definition)[5].

We recall very briefly from [10] that the action (2.1) + (2.3) was obtained by uplifting to eleven dimensions the action for type-IIA gravitational waves derived in [19] using Matrix String Theory in a weakly curved background, and then going beyond the weakly curved background approximation by demanding agreement with Myers action for D0-branes when the waves propagate along the eleventh direction. In the action for type-IIA waves the circle in which Matrix theory is compactified in order to construct Matrix String theory cannot be decompactified in the non-abelian case [19]. In fact, the action exhibits a U(1) isometry associated to translations along this direction, which by construction is also the direction on which the waves propagate. A simple way to see this is to recall that the last operation in the 9-11 flip involved in the construction of Matrix String theory is a T-duality from fundamental strings wound around the 9th direction. Accordingly, in the action we find a minimal coupling to $g_{\mu 9}/g_{99}$ which is the momentum operator k_μ/k^2 if $k^\mu = \delta^\mu_9$. Therefore, by construction, the action is designed to describe BPS waves with momentum charge along the compact isometric direction. It is important to mention that in the abelian limit, when all dielectric couplings and $\mathrm{U}(N)$ covariant derivatives[6] disappear, the action can be Legendre transformed into one in which the dependence on the isometric direction has been restored. This action is precisely the usual one for a massless particle written in terms of an auxiliary γ metric (see [10] and [19] for the details), where no information remains about the momentum charge carried by the particle.

As we have mentioned in the introduction the action (2.1) + (2.3) has been successfully used in the microscopical study of giant graviton configurations in backgrounds which

[4]These couplings are not shown explicitly because they will not play a role in the backgrounds under consideration in this paper.

[5]The reduced metric $\mathcal{G}_{\mu\nu}$ appearing in (2.2) is in fact defined such that its pull-back with ordinary derivatives equals the pull-back of $g_{\mu\nu}$ with these covariant derivatives.

[6]Which are of course implicit in the pull-backs of the action.

are not linear perturbations to Minkowski, like the M-theory backgrounds $AdS_4 \times S^7$ and $AdS_7 \times S^4$ [10, 14]. In all cases perfect agreement with the description of [4, 17] has been found in the limit of large number of gravitons, in which the commutative configurations of [4, 17] become an increasingly better approximation to the non-commutative microscopical configurations [15].

In the next section we will use the same action to describe gravitational waves propagating in the maximally supersymmetric pp-wave background of M-theory. In order to find the quadrupolar coupling to the 6-form potential of this background we will need to interchange the direction of propagation of the waves (the Killing direction in the action) with a compact direction that has to do with the U(1) decomposition of the 5-sphere contained in the background as an S^1 bundle over the two dimensional complex projective space, CP^2.

3. The BMN Matrix model with coupling to the 6-form potential

Let us start by recalling the form of the maximally supersymmetric pp-wave background of M-theory [25, 2]. Starting with the $AdS_4 \times S^7$ background[7], written as:

$$\begin{aligned} ds^2 &= L^2(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_2^2) + 4L^2(d\theta^2 + \cos^2\theta d\phi^2 + \sin^2\theta d\Omega_5^2)\,, \\ C^{(3)}_{\tau\alpha_1\alpha_2} &= -L^3\sinh^3\rho\sqrt{g_\alpha}\,, \qquad C^{(6)}_{\phi\gamma_1\ldots\gamma_5} = -(2L)^6\sin^6\theta\sqrt{g_\gamma}\,; \end{aligned} \tag{3.1}$$

where L is the radius of curvature of AdS_4, $\{\alpha_i\}$ ($\{\gamma_i\}$) are the angle variables parametrizing the 2-sphere (5-sphere) in the notation

$$d\Omega_n^2 = d\beta_1^2 + \sin^2\beta_1(d\beta_2^2 + \sin^2\beta_2(\cdots + \sin^2\beta_{n-1}d\beta_n^2))\,, \tag{3.2}$$

and $\sqrt{g_\beta}$ is the volume element on the unit n-sphere; and defining

$$x^+ = \frac{3}{2\mu}(\tau + 2\phi)\,, \qquad x^- = \frac{\mu L^2}{6}(\tau - 2\phi)\,, \qquad \rho = \frac{r}{L}\,, \qquad \theta = \frac{y}{2L}\,, \tag{3.3}$$

the M-theory maximally supersymmetric pp-wave background is obtained in the limit $L \to \infty$ [26, 1]:

$$\begin{aligned} ds^2 &= -4dx^+dx^- - \left[\left(\frac{\mu}{3}\right)^2 r^2 + \left(\frac{\mu}{6}\right)^2 y^2\right](dx^+)^2 + dr^2 + r^2 d\Omega_2^2 + dy^2 + y^2 d\Omega_5^2 \\ &= -4dx^+dx^- - \left[\left(\frac{\mu}{3}\right)^2 (x_1^2 + x_2^2 + x_3^2) + \left(\frac{\mu}{6}\right)^2 y^2\right](dx^+)^2 + \\ &\quad + dx_1^2 + dx_2^2 + dx_3^2 + dy^2 + y^2 d\Omega_5^2\,, \\ C^{(3)}_{+ij} &= \frac{\mu}{3}\epsilon_{ijk}x^k\,, \qquad i,j = 1,2,3\,, \qquad C^{(6)}_{+\gamma_1\ldots\gamma_5} = -\frac{\mu}{6}y^6\sqrt{g_\gamma}\,, \end{aligned} \tag{3.4}$$

where (x_1, x_2, x_3) parametrize a point in $\mathbb{R}^3$.[8]

[7] One can also start from the $AdS_7 \times S^4$ background and perform a similar limit.

[8] We have not chosen Cartesian coordinates in the $\mathbb{R}^6$ part, as in [1], because for our purposes it will be more convenient to describe the 5-sphere as an S^1 bundle over CP^2, as we discuss below.

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The BMN Matrix model gives the dynamics of DLCQ M-theory in this background along the direction $x^- \sim x^- + 2\pi R$, in the sector with momentum $2p^+ = -p_- = N/R$ [1]. It is a U(N) Matrix theory sum of the usual Matrix theory of [27], a term adding mass to the scalar and fermion fields and a dielectric coupling to the 3-form potential. In this section we are going to show that the same Matrix model, plus a coupling to the six-form potential, arises from the actions (2.1) and (2.3) when the waves propagate in the background (3.4) along the x^- direction.

As we have mentioned above, to do this it is convenient to describe the 5-sphere as an S^1 bundle over CP^2, and introduce adapted coordinates to the U(1) isometry associated to the S^1:

$$d\Omega_5^2 = (d\chi - A)^2 + ds^2_{CP^2}\,, \tag{3.5}$$

with A the connection providing the necessary twist in the fibre to obtain the S^5 as the global space.

CP^2 is the coset manifold SU(3)/ U(2), and it is most conveniently described for our purposes as a submanifold of $\mathbb{R}^8$, determined by a set of four independent constraints (see for instance [28]):

$$\begin{aligned} \sum_{a=1}^{8} z_a z_a &= 1 \\ \sum_{b,c=1}^{8} d^{abc} z_b z_c &= \frac{1}{\sqrt{3}} z_a\,, \end{aligned} \tag{3.6}$$

where $\{z_1, \ldots, z_8\}$ parametrize a point in $\mathbb{R}^8$, and d^{abc} are the components of the totally symmetric SU(3)-invariant tensor defined by

$$\lambda^a \lambda^b = \frac{2}{3}\delta^{ab} + (d^{abc} + i f^{abc})\lambda^c\,, \tag{3.7}$$

where λ^a, $a = 1, \ldots, 8$ are the Gell-Mann matrices.

As we have discussed in the introduction we can construct a *non-perturbative* action for M-theory waves by taking $k^\mu = \delta^\mu_\chi$ in the actions (2.1), (2.3), with χ the coordinate adapted to the U(1) fibre in (3.5). Using this coordinate and the Cartesian coordinates $\{z_1, \ldots, z_8\}$ embedding the CP^2 in $\mathbb{R}^8$ the background metric and potentials read

$$\begin{aligned} ds^2 &= -4dx^+dx^- - \left[\left(\frac{\mu}{3}\right)^2 (x_1^2 + x_2^2 + x_3^2) + \left(\frac{\mu}{6}\right)^2 y^2\right](dx^+)^2 + \\ &\quad + dx_1^2 + dx_2^2 + dx_3^2 + dy^2 + y^2[(d\chi - A)^2 + dz_1^2 + \cdots + dz_8^2]\,, \\ C^{(3)}_{+ij} &= \frac{\mu}{3}\epsilon_{ijk}x^k\,, \qquad i,j = 1,2,3\,, \\ C^{(6)}_{+\chi abcd} &= \frac{\mu}{3} y^6 f^{[abe}f^{cd]f} z^e z^f\,, \qquad a,b,c,d = 1, \ldots 8\,, \end{aligned} \tag{3.8}$$

where f^{abc} are the structure constants of SU(3). Note that the choice of adapted coordinates to the U(1) isometry in the decomposition of the 5-sphere as an S^1 bundle over CP^2 has reduced the explicit invariance of the 5-sphere from SO(6) to SU(3) $\times$ U(1).[9] Therefore

[9]The whole invariance under SO(6) should however still be present in a non-manifest way.

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the background is manifestly invariant under $U(1)^2 \times SO(3) \times SU(3) \times U(1)$, where the first $U(1)^2$ is associated to the translations along the x^- and x^+ directions. The second direction will be identified with the worldline time, taking light-cone gauge $x^+ = t$. The first direction is in turn taken as the direction of propagation of the waves, and will be a commutative direction in the action. Also, y, the radius of the 5-sphere, is taken to be commutative, consistently with the invariance of the background.

It is straightforward to substitute in the CS action (2.3). One finds

$$S^{\rm CS} = -\frac{\mu}{3}\int dx^+\,{\rm STr}\left\{-i\epsilon_{ijk}X^kX^jX^i + \frac{1}{2}y^6 f_{[abe}f_{cd]f}Z^dZ^cZ^bZ^aZ^eZ^f\right\}, \tag{3.9}$$

where we have denoted with capital letters the non-commutative transverse scalars. Similarly, one finds for the BI part:

$$S^{\rm BI} = -\int dx^+\,{\rm STr}\left\{\frac{1}{y}\sqrt{\beta + 4\dot{x}^- - \dot{X}^2 - \dot{y}^2 - y^2\dot{Z}^2}\left(\mathbb{1} - \frac{y^2}{4}[X,X]^2 - \frac{y^6}{4}[Z,Z]^2\right)\right\}, \tag{3.10}$$

where, in our notation $[X,X]^2 \equiv [X^i,X^j][X^i,X^j]$, $[Z,Z]^2 \equiv [Z^a,Z^b][Z^a,Z^b]$,

$$\beta = \left[\left(\frac{\mu}{3}\right)^2(X_1^2 + X_2^2 + X_3^2) + \left(\frac{\mu}{6}\right)^2 y^2\right] \tag{3.11}$$

and $\mathbb{1} - \frac{y^2}{4}[X,X]^2 - \frac{y^6}{4}[Z,Z]^2$ arises as the expansion of the square root of the determinant of Q up to fourth order in the embedding scalars,[10] for Q given by:

$$\begin{aligned} Q^i_j &= \delta^i_j + iy[X^i,X^j]\,, \qquad && i,j = 1,2,3 \\ Q^a_b &= \delta^a_b + iy^3[Z^a,Z^b]\,, && a,b = 1,\dots,8 \\ Q^i_a &= Q^a_i = 0 && \forall\, i,a\,. \end{aligned} \tag{3.12}$$

In order to obtain this expression we have taken the commutators $[X^i,Z^a] = 0$ for all i,a, consistently with the $SO(3) \times SU(3) \times U(1)$ invariance of the background (3.8). In fact, the most general non-commutative ansatz compatible with this symmetry is to take

$$X^i = \frac{r}{\sqrt{C_N}}J^i\,, \qquad i = 1,2,3\,, \tag{3.13}$$

where the J^i form an $N\times N$ representation of $SU(2)$ (in our conventions $[J^i,J^j] = 2i\epsilon^{ijk}J^k$) and C_N is the quadratic Casimir in this representation, and

$$Z^a = \frac{1}{\sqrt{C_N}}T^a\,, \qquad a = 1,\dots,8\,, \tag{3.14}$$

where the T^a form an $N\times N$ representation of $SU(3)$ (in our conventions $[T^a,T^b] = if^{abc}T^c$), and C_N is the quadratic Casimir of $SU(3)$ in this representation. With this ansatz:

$$\sum_{i=1}^{3}(X^i)^2 = r^2\mathbb{1}\,, \qquad \text{and} \qquad \sum_{a=1}^{8}(Z^a)^2 = \mathbb{1}\,, \tag{3.15}$$

[10]The same approximation is taken inside the square root in (3.10). Note that this is the usual approximation taken in non-abelian BI actions [15], which is valid when the non-abelian action is good to describe the system of waves, that is, when the waves are distances away less than the Planck length (in our units $l_p = (\sqrt{2\pi})^{-1}$.

which are the non-commutative analogues of $\sum_{i=1}^{3}(x^i)^2 = r^2$, which is satisfied by (x_1, x_2, x_3) in (3.4), and the first constraint in (3.6) respectively[11]. Then β, $\dot{X}^2$ and $\dot{Z}^2$ are multiples of the identity matrix. We should stress however that this is not the most general non-commutative ansatz one could consider.

Since we are interested in the sector of the theory with a fixed value of the light-cone momentum p_- it is most adequate to perform a Legendre transformation from $\dot{x}^-$ to p_-. Having all terms inside the square root in (3.10) as multiples of the identity matrix, as with the ansätze above, simplifies a lot this transformation, since the square root can be taken out of the symmetrized trace. We will refer to the resulting function as the hamiltonian, though we will not Legendre transform the non-commutative transverse scalars. Calling

$$M \equiv \mathrm{STr}\left(1\!\!1 - \frac{y^2}{4}[X,X]^2 - \frac{y^6}{4}[Z,Z]^2\right) \tag{3.16}$$

we find

$$p_- = -\frac{2M}{y\sqrt{\beta + 4\dot{x}^- - \dot{X}^2 - \dot{y}^2 - y^2\dot{Z}^2}}\,. \tag{3.17}$$

However, before we make a clear statement about the non-commutativity that the embedding scalars should satisfy it is important to recall that the action (2.1) + (2.3) describes waves which, by construction, carry N units of momentum charge in the χ direction. Therefore, this momentum charge will have to be set to zero if we want to describe the sector of the theory with, only, light-cone momentum.

The difference between p_χ being zero or not is merely a coordinate transformation, a boost in χ. However, how to perform coordinate transformations in non-abelian actions is an open problem [29 – 32]. This problem was already encountered in reference [14], in the microscopical study of 5-sphere giant gravitons in terms of dielectric gravitational waves. A careful study on how this limit should be taken can be found there. An essential ingredient is the comparison with the complementary description of the 5-sphere giant graviton configurations in terms of classical M5-branes, and an important check of the validity of the prescription given there is the exact agreement between the two descriptions in the limit of large number of gravitons.

The classical M5 branes of reference [14] were wrapped branes, which were assumed to have momentum in both the x^- and χ directions. The easiest way to obtain a wrapped brane is to consider the uplift to 11 dimensions of a D4 brane. Therefore, the worldvolume of the resulting M5 brane is effectively 4-dimensional, and contains a worldvolume Born-Infeld vector field. While the momentum in the x^- direction is given by letting $x^- = x^-(t)$, the momentum in the internal isometry is given by the coupling of a momentum operator to the Born-Infeld vector. This BI vector is then giving momentum charge, and its instanton number represents the number of momentum units the brane is carrying along the isometric direction. The advantage of this construction is that now we have a clear interpretation of

[11] We will see in the next section that in order to fulfil all the constraints (3.6) we will have to take the Z^a in specific $N \times N$ representations.

how to set to zero p_χ: simply put to zero the instanton number on the brane. Then, by comparing this macroscopical action with its microscopical counterpart, we can learn how to set to zero the microscopical momentum charge along the direction of the isometry.

In the microscopical description the momentum charge is given by the number of gravitational waves of the configuration, namely N. Therefore, naively setting p_χ to zero would imply putting N to zero, which would apparently lead to an ill-defined action. However, a careful comparison with the macroscopical computation shows that this is not the case, and that in the limit $N \to 0$ the remaining action is finite (see [14] for the details). The basic recipe that is derived from the analysis in [14] is that only those terms that remain finite when $N \to 0$ must be kept in the hamiltonian. However, note that to obtain this limit one has to know the non-commutativity that the embedding scalars satisfy, since there are powers of N implicit in the quadratic Casimirs of the groups involved in the non-commutative definitions (see for instance (3.13) and (3.14)). Therefore, to continue further it is necessary to specify the non-commutativity that the transverse scalars satisfy.

Taking the most general ansatz compatible with the symmetry of the background, namely (3.13) and (3.14), one obtains the hamiltonian

$$H = \int dx^+ \Big\{ \frac{p_-}{2}\left(\frac{1}{2}(\dot{X}^2 + \dot{y}^2 + y^2\dot{Z}^2) - \frac{\beta}{2} - \frac{2M^2}{y^2 p_-^2}\right) - i\frac{\mu}{3}\epsilon_{ijk}X^k X^j X^i + \\ + \frac{\mu}{6}y^6 f_{[abe}f_{cd]f} Z^d Z^c Z^b Z^a Z^e Z^f \Big\}. \tag{3.18}$$

Taking into account that $p_- = -N/R$ and keeping only those terms that remain finite when $N \to 0$, as explained above, we have that

$$\frac{M^2}{y^2 p_-} \to \frac{1}{2}R[X,X]^2 - \frac{1}{16}Ry^{10}[Z,Z]^4\,. \tag{3.19}$$

We then arrive at

$$H = -\int dx^+\, \mathrm{STr}\Big\{ \frac{1}{4R}\left(\dot{X}^2 + \dot{y}^2 + y^2\dot{Z}^2\right) - \frac{1}{4R}\left(\frac{\mu^2}{9}X^2 + \frac{\mu^2}{36}y^2\right) + \frac{1}{2}R[X,X]^2 - \\ - \frac{1}{16}R\,y^{10}[Z,Z]^4 + i\frac{\mu}{3}\epsilon_{ijk}X^k X^j X^i - \\ - \frac{\mu}{6}y^6 f_{[abe}f_{cd]f} Z^d Z^c Z^b Z^a Z^e Z^f \Big\}, \tag{3.20}$$

where we have made use of expressions (4.2) and (4.3), which are included in section 4, where we discuss in more detail the non-commutativity of the fuzzy 5-sphere.

Comparing with the BMN Matrix model we find the same expression in the $\mathbb{R}^3$ part of the geometry, where we are using the same Cartesian coordinates (X_1, X_2, X_3). Regarding the $\mathbb{R}^6$ part, BMN use Cartesian coordinates, in which the SO(6) invariance is manifest, whereas we are using coordinates in which this invariance is reduced to SU(3) $\times$ U(1), and, moreover, we have implicitly taken a particular non-commutative ansatz for these coordinates. It could be that, apart from the new dielectric coupling involving the Z^a coordinates, both actions were equivalent through the right coordinate transformation, although we have not checked this out explicitly. In any case a non-trivial check of the validity of our hamiltonian (3.20) is that we will find a new fuzzy 5-sphere solution with the correct radius in the large N limit.

In the next section we will see how the fuzzy 5-sphere solution arises. However, before doing that we are going to derive the fuzzy 2-sphere solution of [1] in our notation, and we are going to compare it with the calculation in terms of a classical spherical M2-brane that we have included in the appendix A for the sake of completeness.

The BMN Matrix model contains a fuzzy 2-sphere solution with non-zero momentum p_- and zero light-cone energy, which is the microscopical realization of the classical 2-sphere giant graviton of [4] in the pp-wave background [1]. Taking the ansatz $r = \text{const}$, $y = Z^a = 0$, $a = 1, \dots 8$, and (3.13) for (X_1, X_2, X_3) in (3.20), we arrive at the following hamiltonian

$$H = \int dx^+ \frac{N}{R} r^2 \left(\frac{\mu}{6} - \frac{2Rr}{\sqrt{N^2-1}} \right)^2 . \tag{3.21}$$

Minimizing with respect to r one finds a zero energy solution for $r = 0$, which corresponds to the point-like graviton, and another one for finite r:

$$r = \frac{\mu}{12} \frac{\sqrt{N^2-1}}{R} , \tag{3.22}$$

which corresponds to the giant graviton solution. When the number of gravitons is large this value of the radius agrees with the radius of the classical 2-sphere giant graviton solution in this background, given by expression (A.6) in the appendix A (recall that $T_2 = (2\pi)^{-1}$ in our units, in which T_0, the tension of a single graviton, is equal to 1, and that we are describing the sector of the theory with $p_- = -N/R$). Moreover, there is perfect agreement between the corresponding hamiltonians, given by (3.21) and (A.5), in this limit.

In the next section we will show that the extra dielectric term present in our action (3.20) is responsible for the existence of a second zero energy solution which is interpreted as the fuzzy 5-sphere giant graviton.

4. The fuzzy 5-sphere giant graviton

It has been shown in [14] that the 5-sphere giant graviton solutions in $AdS_4 \times S^7$ and $AdS_7 \times S^4$ are realized microscopically in terms of fuzzy 5-spheres defined as S^1 bundles over fuzzy CP^2.

The fuzzy CP^2 has been extensively studied in the literature, in different contexts [28, 33, 38]. Embedding the CP^2 in $\mathbb{R}^8$ as in section 3 the simplest way to construct a fuzzy CP^2 is by making the z_a coordinates non-commutative and impose the constraints (3.6) at the level of matrices. One sees that taking

$$Z^a = \frac{1}{\sqrt{C_N}} T^a \tag{4.1}$$

with T^a the generators of SU(3) in an N dimensional representation, and C_N the quadratic Casimir in this representation, the constraints are satisfied iff the Z^a are taken in the $(n,0)$ or $(0,n)$ representations of SU(3) [28], parametrizing the irreducible representations of SU(3) by two integers (n,m) corresponding to the number of fundamental and anti-fundamental indices (see [14] and references therein for more details). In these representations

$$Z^a = \frac{1}{\sqrt{\frac{1}{3}n^2+n}} T^a \,, \tag{4.2}$$

and the commutation relations of the Z^a become

$$[Z^a, Z^b] = \frac{i}{\sqrt{\frac{1}{3}n^2+n}} f^{abc} Z^c \,. \tag{4.3}$$

The fuzzy 5-sphere solution emerges when, on top of (4.2), $r = X^i = 0$, $i = 1,2,3$, and y and Z^a are taken time independent in the hamiltonian (3.20). Then one gets

$$H = \int dx^+ \frac{N}{R} y^2 \left(\frac{\mu}{12} - \frac{Ry^4}{4(n^2+3n)} \right)^2 , \tag{4.4}$$

where we have used that

$$f_{[abe} f_{cd]f} Z^d Z^c Z^b Z^a Z^e Z^f = -\frac{1}{4(n^2+3n)} \mathbb{1} \,. \tag{4.5}$$

Minimizing the hamiltonian (4.4) one finds two zero light-cone energy solutions: one for $y = 0$, the point-like graviton, and another one for

$$y = \left(\frac{\mu(n^2+3n)}{3R} \right)^{1/4} , \tag{4.6}$$

which corresponds to the giant graviton solution. Taking into account that the dimension of the $(n,0)$ and $(0,n)$ representations is given by

$$N = \frac{(n+1)(n+2)}{2} \tag{4.7}$$

and that $p_- = -N/R$, we have that for large N

$$y \sim \left(-\frac{2}{3}\mu p_- \right)^{1/4} \tag{4.8}$$

which is the radius of the classical 5-sphere solution discussed in the appendix A, where we have to take into account that $T_5 = (8\pi^3)^{-1}$ in our units. Also, in this limit, the hamiltonians (4.4) and (A.5) agree exactly. This agreement is an important check for the validity of our Matrix model (3.20).

5. Conclusions

Using the action for N coincident gravitational waves in M-theory we have constructed a pp-wave Matrix model containing the fuzzy 5-sphere giant graviton as a supersymmetry preserving solution. Our Matrix model is the sum of the (bosonic part of the) BMN Matrix action[12] plus an additional quadrupolar coupling to the 6-form potential of the pp-wave

[12] With the remarks that we have made after equation (3.20).

background. This coupling is the one responsible for the expansion of the waves into a fuzzy 5-sphere. We have seen that in order to see this new coupling arising it is necessary to go to a more non-perturbative regime in M-theory, in which the waves are related to type-IIA waves upon reduction. In this regime the fuzzy 5-sphere giant graviton arises as one of the minima of the potential, together with the point like graviton solution. Different approaches are considered in the literature which tend to clarify which of the two minima corresponds to the true vacuum, and, in $AdS_m \times S^n$ spacetimes, whether or not the stringy exclusion principle can be satisfied (see for instance [4, 17, 18]). This analysis is however beyond the scope of our paper.

We would like to stress that using our action (3.20) the point-like and fuzzy 5-sphere graviton solutions arise as minima of the potential in exactly the same way the point-like and fuzzy 2-sphere gravitons arise from the BMN action. As we have discussed this is possible because our action describes a more non-perturbative regime of M-theory. In the more perturbative regime of the BMN matrix model, the 5-brane giant graviton would be reproduced as excited states about the trivial vacuum (the point-like solution) [20]. As it has been pointed out in this reference, the linear fluctuations around the $X = 0$ vacuum of the BMN action match with the protected quantities for a spherical M5-brane. Therefore this vacuum is identifyed with the 5-sphere giant graviton, and it is argued that non-perturbative effects would blow up the M5-brane out of the $X = 0$ vacuum. This was already conjectured in [1]. However, in our situation, since we are in a more non-perturbative regime, our action contains as a vacuum solution the M5-brane. Thus, the trivial $X = 0$ solution truly corresponds in our matrix model to the point-like graviton.

The situation above is very similar to the way the expansion of N D3-branes into an NS5-brane in mass deformed $\mathcal{N} = 4$ SYM [21] is realized perturbatively. This expanded configuration is the supergravity dual of the confining vacuum of the gauge theory, and can be reproduced perturbatively as excited states about the trivial vacuum [21]. On the other hand at strong coupling the description can be made in terms of a non-perturbative action for N D3-branes, which shows a dielectric coupling that is responsible for the expansion of the branes into an NS5-brane [39]. As predicted by S-duality, this vacuum is described at strong coupling in exactly the same way the expansion of the D3-branes into a D5-brane, corresponding to the Higgs vacuum of the gauge theory, is described at weak coupling.

We should emphasize that we have only worked out the bosonic terms in the action. However the agreement between our fuzzy 2- and 5-sphere solutions and the classical description in [4] suggests that both should occur as BPS solutions of a supersymmetric Matrix action, preserving the same half of the supersymmetries as the point-like graviton [17]. It would be interesting to see if this Matrix action could be derived as in [1] keeping the 6-form potential of the background and using the supervielbeins for $AdS_4 \times S^7$ and $AdS_7 \times S^4$ of [40].

A Matrix action having a fuzzy 5-sphere giant graviton solution has also been given recently in [24]. The approach taken in this reference is to construct the Matrix action such that the defining algebra of the fuzzy 5-sphere of [22] is obtained from its equations of motion. The fuzzy 5-sphere defined in [22] is however different from the fuzzy 5-sphere that we have obtained in this paper, as solution to our Matrix model. One obvious difference is that the fuzzy S^5 in [22] has explicit SO(6) invariance whereas our construction is only SU(3) $\times$ U(1) manifestly invariant.[13] Another difference is that our solution approaches neatly the classical S^5 in the large N limit, where all the non-commutativity disappears, whereas this is not the case for the fuzzy 5-sphere of [22]. Other more technical differences can be found in [14]. Therefore there are clear differences between both fuzzy sphere constructions. From this point of view one should not expect exact coincidence between the Matrix model presented in [24] and our Matrix model.

We believe one nice feature about our Matrix model is that it reproduces the right value for the radius of the 5-sphere giant graviton solution. This value cannot however be predicted from the Matrix model in [24], because the requirement of reproducing the defining algebra of the fuzzy 5-sphere of [22] from its equations of motion is not enough to fix completely all the parameters in the action. Moreover, some arguments suggest that the radius of this giant graviton solution should scale with the light-cone momentum as $r \sim (p_+)^{1/5}$ [41, 23], so it would not reproduce the correct value for the classical 5-sphere found in [4, 20] in the large N limit. The description of the fuzzy 5-sphere solution as an S^1 bundle over a fuzzy CP^2 seems to be an essential ingredient towards the correct increasing of the power of the radius of the solution with the light-cone momentum.

Our Matrix model arises from the action for coincident M-theory gravitational waves constructed in [10]. Using dualities similar actions have been derived for type-II gravitational waves (see [19, 12]). For consistency the pp-wave Matrix models, constructed in the literature using different approaches, in type IIA [42 − 45] and type IIB [23] should also arise from these actions when the waves propagate in the pp-wave background with non-vanishing light-cone momentum. From this point of view the physical interpretation of the fundamental constituents referred as *tiny gravitons* in references [23, 46] would be as gravitational waves, and, clearly, the theory describing them would be a U(N) gauge theory, since these actions arise through dualities from the U(N) gauge theory describing M-theory gravitational waves. These dualities imply in particular that the vector field in the type-IIA Matrix model would be associated to D2-branes wrapped on the direction of propagation of the waves, ending on them, and in the type-IIB Matrix model would be associated to D3-branes wrapped on the direction of propagation and also on a second compact direction (the T-duality direction from type-IIA) ending on them. We hope to report the details of these calculations in a forthcoming paper [47].

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[13]The SO(6) invariance might still be present in a non-manifest way, in the same way the classical S^5 is not explicitly SO(6) invariant when it is described as an S^1 bundle over CP^2.

A. Macroscopical giant gravitons for maximally supersymmetric pp-wave backgrounds

In this appendix we summarize the macroscopical description of the giant graviton solutions for the pp-wave backgrounds that arise as Penrose limits of $AdS_m \times S^n$ (see [20, 45, 48]).

The Penrose limits of the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ spacetimes give rise to the background (3.4), whereas that of $AdS_5 \times S^5$ gives rise to [49]:

$$\begin{aligned} ds^2 &= -4dx^+dx^- - \mu^2(x_1^2 + \cdots + x_8^2)(dx^+)^2 + d\vec{x}^2\,, \qquad \text{where } \vec{x} \in \mathbb{R}^8 \\ C^{(4)}_{+\alpha_1\alpha_2\alpha_3} &= -\mu r^4 \sqrt{g_\alpha}\,, \qquad\qquad C^{(4)}_{+\gamma_1\gamma_2\gamma_3} = \mu y^4 \sqrt{g_\gamma}\,, \end{aligned} \tag{A.1}$$

where $\{\alpha_i\}$ and $\{\gamma_i\}$ are the angle variables parametrizing the two 3-spheres embedded in $\mathbb{R}^8$, and $r^2 = x_1^2 + \cdots + x_4^2$, $y^2 = x_5^2 + \cdots + x_8^2$.

The giant graviton solutions associated to these backgrounds are spherical 2-branes and 5-branes for the Penrose limit of $AdS_4 \times S^7$ and $AdS_7 \times S^4$, and spherical 3-branes for the background (A.1). In all cases the branes are stable thanks to their dipole or magnetic moment with respect to the background potential.

Taking the ansatz for a p-brane giant graviton solution in these backgrounds one has that, generically

$$ds^2 = -4dx^+dx^- - \beta_p(dx^+)^2 + d\vec{x}^2 \tag{A.2}$$

where $\vec{x}$ parametrizes a point in $\mathbb{R}^{p+1}$, $\beta_p = \sigma_p \mu^2 r^2$ with $\sigma_2 = \frac{1}{9}$, $\sigma_3 = 1$ and $\sigma_5 = \frac{1}{36}$, and r is the radius of the p-brane, $r^2 = x_1^2 + \cdots + x_{p+1}^2$. The non-vanishing background potential can also be written in a unified way, as

$$C^{(p+1)}_{+\alpha_1\ldots\alpha_p} = -\sqrt{\sigma_p}\mu r^{p+1}\sqrt{g_\alpha}\,. \tag{A.3}$$

We fix a gauge in which the spatial coordinates are identified with the angular coordinates and $t = x^+$. The p-brane moves along the x^- direction by taking $x^- = x^-(x^+)$.

Substituting this trial solution into the worldvolume action of the p-brane and integrating the angular worldvolume coordinates one finds

$$S_p = T_p A_p \int dx^+ \left\{ -r^p \sqrt{\sigma_p \mu^2 r^2 + 4\dot{x}^-} + \sqrt{\sigma_p}\mu r^{p+1} \right\} \tag{A.4}$$

where A_p is the area of a unit p-sphere.

Legendre transforming $\dot{x}^-$ to p_- one finally arrives at a hamiltonian

$$H = -\int dx^+ p_- r^2 \left(\frac{1}{2}\sqrt{\sigma_p}\mu + \frac{T_p A_p r^{p-1}}{p_-} \right)^2 \tag{A.5}$$

from which one finds that the light-cone energy vanishes for $r = 0$, which corresponds to the point-like graviton, and for

$$r = \left(-\frac{\sqrt{\sigma_p}\mu p_-}{2T_p A_p} \right)^{1/(p-1)} \tag{A.6}$$

which corresponds to the giant graviton solution.

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